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by Robert E. Fulton and Nancy P. Sykes

Langley Research Center
Langley Station, Hampton, Va.

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SUMMARY

Results are presented for the effect of flexural stiffness of face sheets on the buckling of elastic cylindrical shells of sandwich construction subjected to axial and lateral loading. This study shows that when the core is very weak in shear the flexural stiffness of the face sheets can significantly increase the buckling load. This increase is shown to be much larger for axially loaded shells than for laterally loaded ones. Graphical results are given for a wide range of the significant parameters. The results are based on a linear buckling theory for sandwich shells of the Donnell type, which takes into account the asymmetry of the sandwich faces.

INTRODUCTION

A number of studies have been made over the past few years of the buckling characteristics of cylindrical shells of sandwich construction. In most of these studies, the contribution of the flexural stiffness of the face sheets has been neglected when considering the buckling characteristics of the sandwich. In certain ranges of the significant parameters this assumption is reasonable; however, in the range where large shearing deformations occur in the core at buckling, it may not be reasonable for certain loading conditions. Reference 1 is a detailed parametric study of the contribution of face-sheet bending stiffness to the buckling load of curved sandwich plates in axial compression. The purpose of this paper is to carry out a similar detailed investigation of this contribution for complete cylinders subjected to either axial or lateral loading. The effect of face-sheet stiffness is shown to be an important factor in certain ranges of the parameters and is greater for axial loading than for lateral loading.

SYMBOLS

a,b	length and circumference of cylinder, respectively ($b = 2\pi r$)
B	extensional stiffness of a face sheet, $\dfrac{Et}{1 - \mu^2}$

c	thickness of core
D	flexural stiffness of a face sheet, $\frac{Et^3}{12\left(1 - \mu^2\right)}$
E	Young's modulus for a face sheet
F	Airy stress function
G_c	shear modulus for core
h	distance between middle surfaces of face sheets
k_x	buckling-load coefficient for cylinder in axial compression, $\frac{N_x^* a^2\left(B_1 + B_2\right)}{\pi^2 h^2 B_1 B_2}$
k_y	buckling-load coefficient for cylinder subjected to lateral load, $\frac{N_y^* a^2\left(B_1 B_2\right)}{\pi^2 h^2 B_1 B_2}$
M_x,M_y	sum of the individual moments in the two face sheets in x and y directions, respectively
m,n	integers
N_x^*,N_y^*,N_{xy}^*	stress resultants causing buckling
r	radius to neutral axis of sandwich shell (fig. 1)
S	face-sheet stiffness parameter, $\frac{\left(B_1 + B_2\right)\left(D_1 + D_2\right)}{h^2 B_1 B_2}$
t	thickness of face sheet
u,v,w	buckling displacements of the shell in the x, y, z directions (fig. 1)
x,y,z	coordinates of the neutral surface of shell (fig. 1)
$Z_a{}^2$	curvature parameter for cylinder, $\frac{a^4\left(B_1 + B_2\right)^2\left(1 - \mu^2\right)}{r^2 h^2 B_1 B_2}$
α	buckling rotation in x direction, $\frac{1}{h}\left(u_1 - u_2\right)$

β	buckling rotation in y direction, $\frac{1}{h}\left(v_1 - v_2\right)$
μ	Poisson's ratio
φ	coupled rotation variable, $\alpha,_x + \beta,_y$
ψ_a	sandwich core parameter for cylinder, $\frac{\pi^2 c B_1 B_2}{a^2 G_c \left(B_1 + B_2\right)}$
λ	wave-length ratio, $\frac{an}{bm}$
∇^2	two-dimensional Laplacian operator

Subscripts:

1,2	refer to upper and lower face sheets, respectively
x,y	after commas indicate partial differentiation with respect to the longitudinal and circumferential directions, respectively

BUCKLING EQUATIONS

The configuration to be considered is a circular cylinder of nonsymmetrical sandwich section subjected to a constant lateral force N_y^* or a constant axial force N_x^* (fig. 1). The equations governing the buckling of such a cylinder, taken from references 1 and 2, are

$$\nabla^4 F = -\left(B_1 + B_2\right)\left(1 - \mu^2\right)\frac{w,_{xx}}{r} \tag{1a}$$

$$\left[1 - \frac{cB_1B_2}{G_c\left(B_1 + B_2\right)}\nabla^2\right]\varphi = \nabla^2 w \tag{1b}$$

$$\left(D_1 + D_2\right)\nabla^4 w - N_x^* w,_{xx} - N_y^* w,_{yy} - \frac{1}{r}F,_{xx} + \frac{h^2 B_1 B_2}{B_1 + B_2}\nabla^2\varphi = 0 \tag{1c}$$

In these equations the concept of a sandwich is retained, in that the core undergoes only transverse shear deformations so that a line through the undeformed core remains straight when deformed but does not necessarily remain perpendicular to the neutral surface of the sandwich shell. The core is assumed to be elastic, isotropic, and homogeneous; to carry no inplane loads; and to have no deformation in the direction normal to the

neutral surface of the shell. The total thickness of the shell element is assumed to be small compared with the radius of curvature. The face sheets are assumed to be elastic, isotropic, and homogeneous and to conform to the classical shell theory; that is, to satisfy the Kirchhoff-Love condition. Poisson's ratio is taken to be the same for the two face sheets and the core; however, the thicknesses and Young's modulus can be different.

The simply supported boundary conditions at buckling are defined herein as follows along $x = 0$ and $x = a$:

(1) Displacement normal to the surface of the shell vanishes; that is,

$$w = 0 \tag{2a}$$

(2) Moment couple normal to the edge caused by differential inplane forces in the face sheet vanishes; that is,

$$\alpha_{,x} + \mu\beta_{,y} = 0 \tag{2b}$$

(3) The sum of the moment in each of the individual face sheets vanishes; that is,

$$M_x = -\left(D_1 + D_2\right)\left(w_{,xx} + \mu w_{,yy}\right) = 0 \tag{2c}$$

(4) Displacements parallel to each edge are prevented; that is,

$$v = \frac{B_1 v_1 + B_2 v_2}{B_1 + B_2} = 0 \tag{2d}$$

$$\beta = 0 \tag{2e}$$

(5) Displacements normal to each edge in the neutral surface of the shell occur freely; that is,

$$F_{,yy} = 0 \tag{2f}$$

The solution to equations (1) satisfying the simple-support boundary conditions for N_x^* and N_y^* independent of x and y is (ref. 1):

$$w = w_o \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{3}$$

$$F = \left(B_1 + B_2\right)\left(1 - \mu^2\right)\frac{w_o}{r} \frac{m^2}{\pi^2 a^2\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{4}$$

$$\varphi = -\frac{w_o\pi^2\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)}{1 + \pi^2 \frac{cB_1B_2}{G_c\left(B_1 + B_2\right)}\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{5}$$

where w_o is a constant and m and n are integers. Substitution of equations (3), (4), and (5) into equation (1c) yields the buckling condition:

$$-N_x^*\pi^2 \frac{m^2}{a^2} - N_y^*\pi^2 \frac{n^2}{b^2} + \left(D_1 + D_2\right)\pi^4 \frac{\left(m^2 + \frac{a^2}{b^2} n^2\right)^2}{a^4} + \frac{\left(B_1 + B_2\right)\left(1 - \mu^2\right)m^4}{r^2\left(m^2 + \frac{a^2}{b^2} n^2\right)^2}$$

$$+ \frac{\frac{h^2B_1B_2}{B_1 + B_2}\pi^4\left(m^2 + \frac{a^2}{b^2} n^2\right)^2}{a^4\left[1 + \psi_a\left(m^2 + \frac{a^2}{b^2} n^2\right)\right]} = 0 \tag{6}$$

where

$$\psi_a = \frac{\pi^2 cB_1B_2}{a^2G_c\left(B_1 + B_2\right)}$$

It should be noted that equation (6) holds for all ranges of the parameters and is applicable to both curved plates and cylinders. If $c = 0$, equation (6) reduces to a Donnell type result for a bilayered shell. With $c = 0$ and the face sheets symmetrical, it reduces to that given for an isotropic shell in reference 3.

Axial Compression

Equation (6) has been specialized to apply to the case of constant axial compression and is

$$k_x = Sm^2\left(1 + \lambda^2\right)^2 + \frac{Z_a^2/\pi^4}{m^2\left(1 + \lambda^2\right)^2} + \frac{m^2\left(1 + \lambda^2\right)^2}{1 + \psi_a m^2\left(1 + \lambda^2\right)} \tag{7}$$

Equation (7) is minimized with respect to λ and n, where λ is regarded as a continuous variable and n is an even integer.

The results of these calculations are presented in figure 2. Note that the results for $S = 0$ (i.e., face-sheet flexural stiffness neglected) shown in figure 2 agree with those given in reference 4, and the results in figure 2(a) for $S = 0.3333$ and $c = 0$ $(t_1 = t_2 = h)$ agree with those for a homogeneous cylinder given by Batdorf in reference 3.

Lateral Loading

Equation (6) has been specialized to apply to the case of constant lateral loading and takes the form

$$k_y = \frac{S\left(1 + \lambda^2\right)^2}{\lambda^2} + \frac{{Z_a}^2}{\pi^4 \lambda^2 \left(1 + \lambda^2\right)^2} + \frac{\left(1 + \lambda^2\right)^2}{\lambda^2\left[1 + \psi_a\left(1 + \lambda^2\right)\right]} \tag{8}$$

In equation (8) m is set equal to 1 and the minimization is carried out with respect to λ (i.e., an/b). The results are given for certain ranges of the parameters in figure 3. For $S = 0.3333$ and $c = 0$, the results given in figure 3(a) again agree with those given by Batdorf (ref. 3) for the homogeneous cylinder.

DISCUSSION OF RESULTS

A significant result to be noted from figures 2 and 3 is the effect of the face-sheet stiffness on the buckling strength of cylinders under the two different loading conditions. One important range of practical interest for a sandwich is the range $S < 0.01$, which corresponds to t/h less than about 1/6 for a symmetrical sandwich. Under both axial and lateral loading, the effect of face-sheet stiffness for $S < 0.01$ is not significant for small values of Z_a. On the other hand, for large values of Z_a the face-sheet flexural stiffness significantly increases the buckling load of the shell.

For large values of Z_a, simple approximate buckling formulas which account for face-sheet stiffness may be derived for both axial loading and lateral loading. The results in reference 1 show that for large Z_a the curves for axial loading approach the curve

$$k_x = \frac{1}{\psi_a} + \frac{2Z_a}{\pi^2}\sqrt{S}$$

A similar analysis indicates that the curves for lateral loading for large Z_a approach

$$k_y = \frac{1}{\psi_a} + \sqrt{\frac{2Z_a}{\pi^2}}\left(\frac{4}{3} S\right)^{3/4}$$

These two equations show that for large Z_a the increase in buckling load when S is not zero is proportional to Z_a and $\sqrt{S}$ for axial loading and is proportional to $\sqrt{Z_a}$ and $(S)^{3/4}$ for lateral loading. Because Z_a is large and S is less than 1, the effect of the face-sheet bending stiffness is more pronounced for axial loading than for lateral loading.

CONCLUDING REMARKS

A study has been made of the effect of flexural stiffness of the face sheets on the buckling of elastic cylindrical shells of sandwich construction subjected to axial and lateral loading. Results are given for a wide range of the significant parameters. The results agree at one extreme with classical sandwich theory and at the other extreme with isotropic shell theory. Any face-sheet flexural stiffness is seen to increase the buckling strength of the shell significantly in certain ranges of the shell parameters where the core is weak in shear. Simple buckling formulas are given for these ranges and this increase in buckling strength is much larger for axially loaded cylindrical shells than for laterally loaded ones.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., February 23, 1966.

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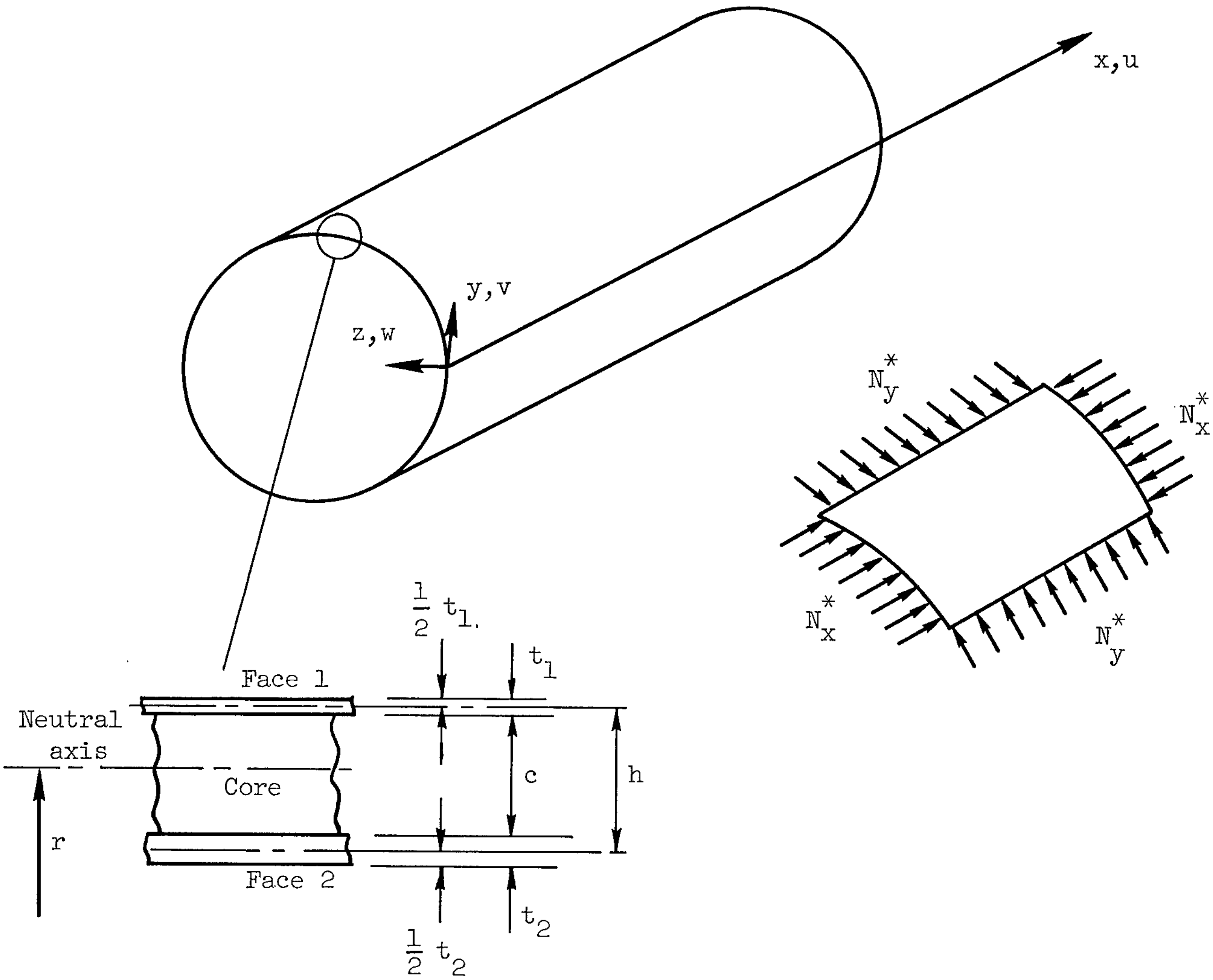

Figure 1.- Cylindrical sandwich shell subjected to axial and lateral loading.

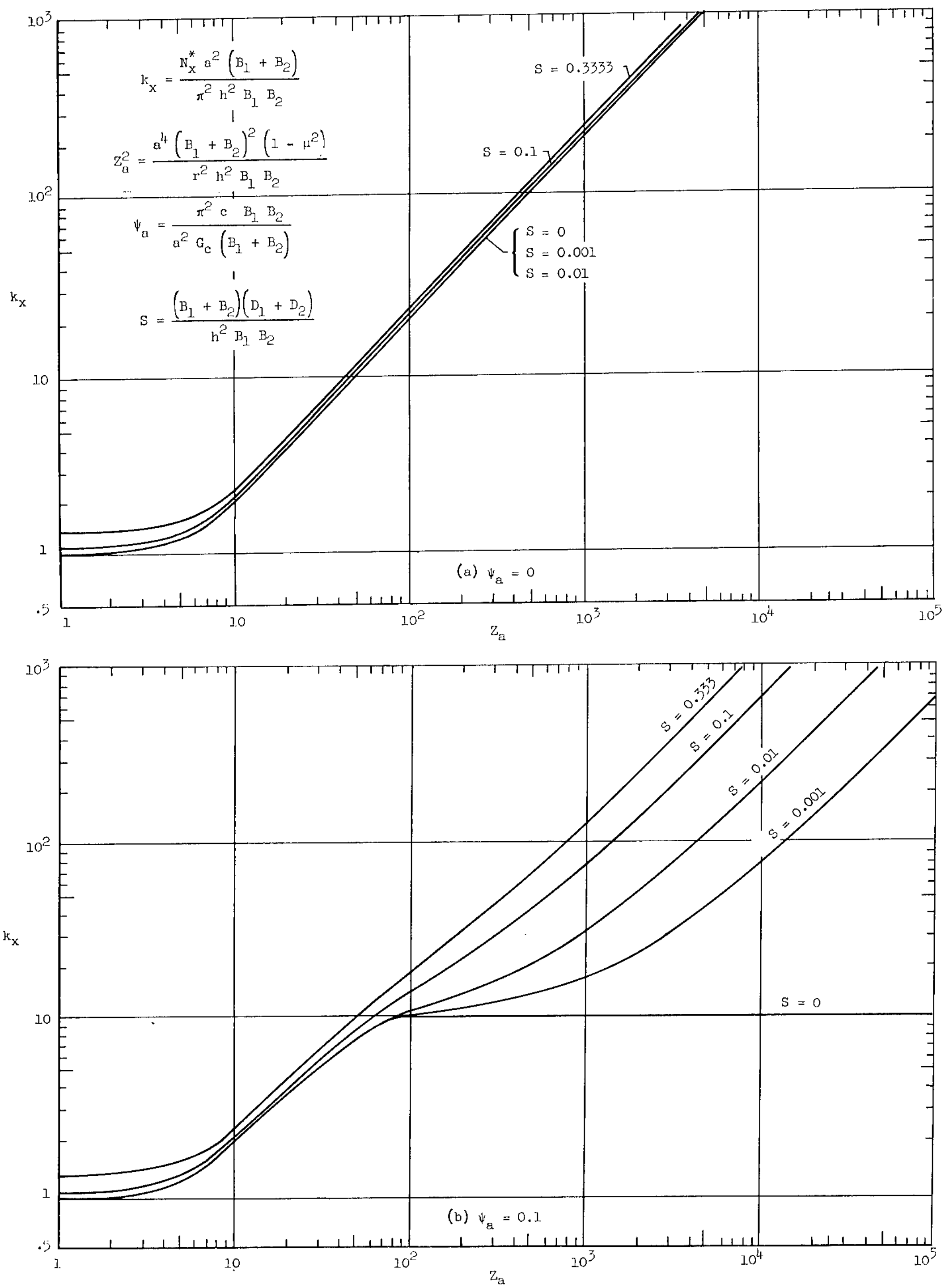

Figure 2.- Buckling coefficients for simply supported cylindrical sandwich shell in axial compression.

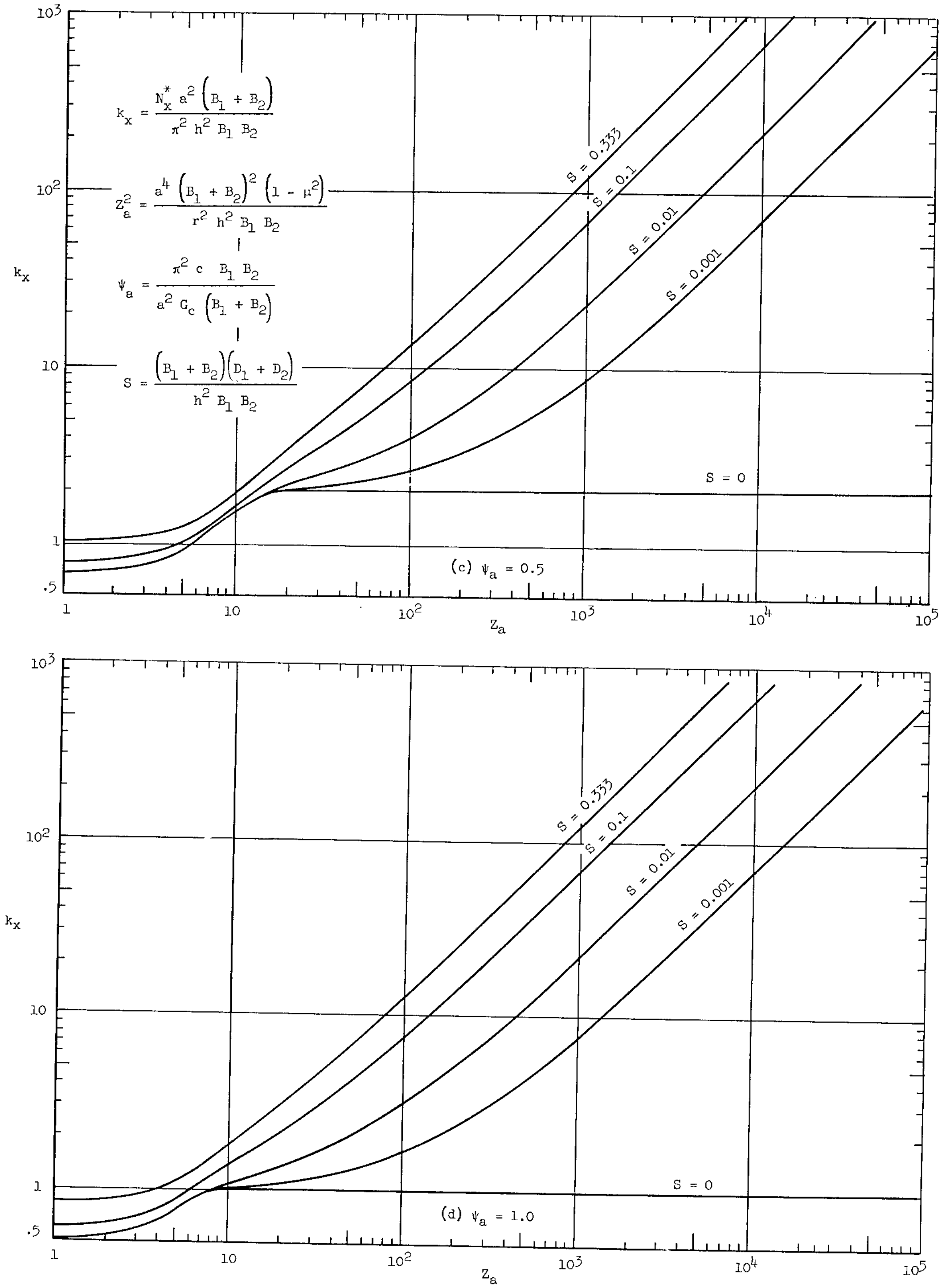

Figure 2.- Concluded.

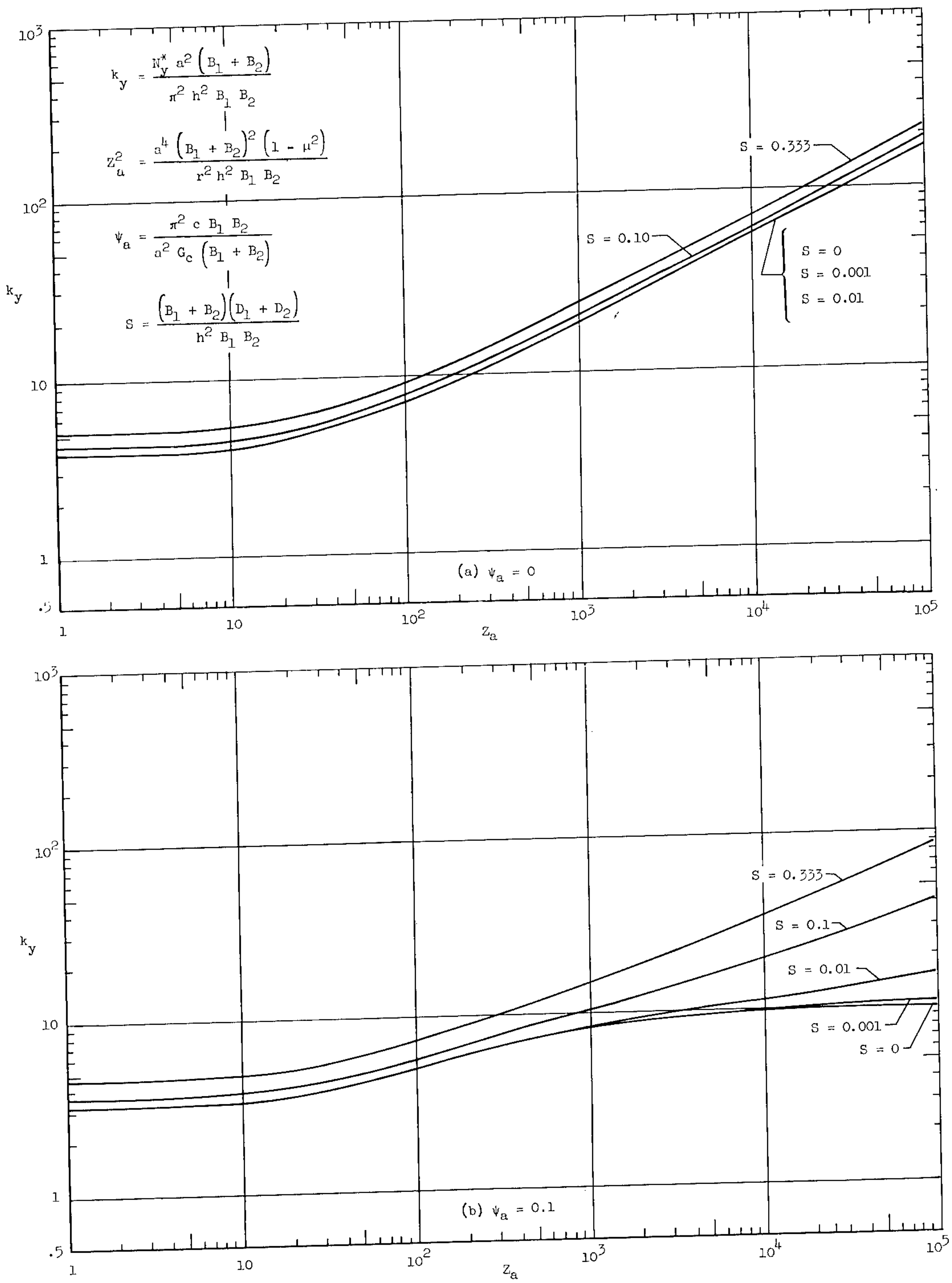

Figure 3.- Buckling coefficients for simply supported cylindrical sandwich shell under lateral loading.

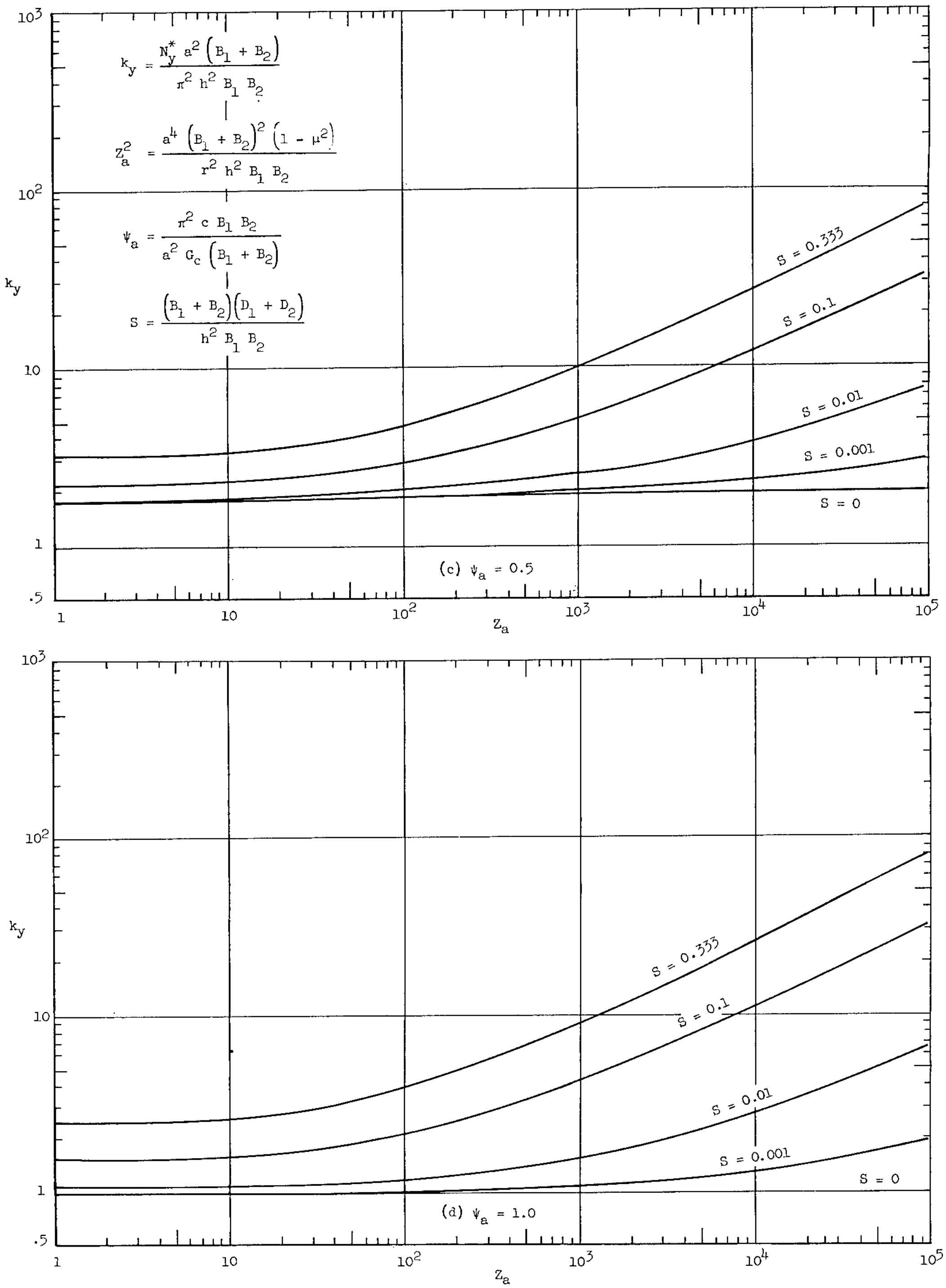

Figure 3.- Concluded.